

Exercise 16

Normalize the vectors in Exercises 6 to 8. (Only the solution corresponding to Exercise 7 is in the Student Guide.)

Solution

In order to normalize a vector, divide it by its magnitude. This results in a unit vector, a vector that has a magnitude of 1.

Exercise 6

$$\mathbf{u} = 15\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}, \mathbf{v} = \pi\mathbf{i} + 3\mathbf{j} - \mathbf{k}$$

$$\|\mathbf{u}\| = \sqrt{15^2 + (-2)^2 + 4^2} = \sqrt{245}$$

$$\|\mathbf{v}\| = \sqrt{\pi^2 + 3^2 + (-1)^2} = \sqrt{\pi^2 + 10}$$

Normalize the vectors.

$$\hat{\mathbf{u}} = \frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{\sqrt{245}}(15\mathbf{i} - 2\mathbf{j} + 4\mathbf{k})$$

$$\hat{\mathbf{v}} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{\sqrt{\pi^2 + 10}}(\pi\mathbf{i} + 3\mathbf{j} - \mathbf{k})$$

Exercise 7

$$\mathbf{u} = 2\mathbf{j} - \mathbf{i}, \mathbf{v} = -\mathbf{j} + \mathbf{i}$$

$$\|\mathbf{u}\| = \sqrt{2^2 + (-1)^2} = \sqrt{5}$$

$$\|\mathbf{v}\| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$$

Normalize the vectors.

$$\hat{\mathbf{u}} = \frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{\sqrt{5}}(2\mathbf{j} - \mathbf{i})$$

$$\hat{\mathbf{v}} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{\sqrt{2}}(-\mathbf{j} + \mathbf{i})$$

Exercise 8

$$\mathbf{u} = 5\mathbf{i} - \mathbf{j} + 2\mathbf{k}, \quad \mathbf{v} = \mathbf{i} + \mathbf{j} - \mathbf{k}$$

$$\|\mathbf{u}\| = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{30}$$

$$\|\mathbf{v}\| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$$

Normalize the vectors.

$$\hat{\mathbf{u}} = \frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{\sqrt{30}}(5\mathbf{i} - \mathbf{j} + 2\mathbf{k})$$

$$\hat{\mathbf{v}} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} - \mathbf{k})$$